

COLLEGE PROBABILITY AND STATISTICS

HOMEWORK

Dr. John Loase

Principal Investigator National Science Foundation
Program Mathematical (Statistical) Modeling

Edited by Zhaojing (Paris) Shen

COLLEGE PROBABILITY AND STATISTICS

PART 1 Probability

Topics 1-11

Pages 1-25

Key statistical formulas

1. mean = $\frac{\sum x}{n}$

2. median = middle number (average if there are two middle numbers)

3. variance = $\sigma^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}$

4. standard deviation = $\sigma = \sqrt{\text{variance}}$ coef of var = $\frac{s.dev}{mean} * 100\%$

5. r = correlation coef. = $\frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$

6. P(A or B) --- single event

a. If A, B are mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

b. If A, B overlap

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

7. P(A and B) --- 2 or more consecutive events

$$P(A \text{ and } B) = P(A) * P(B|A)$$

P(B|A) = probability of B if A occurs

8. counting --- see sheet for 3 formulas

Permutations

Combinations

Counting principle

9. Binomial probability

N = total number

X = number of successes

P = probability of success

$$P(x \text{ success}) = c(n, x) p^x (1-p)^{n-x}$$

10. Mean/ st. Deviation of a random variable

$$\mu = \sum x * p(x)$$

$$\sigma^2 = \sum (x - \mu)^2 * p(x)$$

$$68\%CI = (\mu - 1\sigma, \mu + 1\sigma)$$

$$96\%CI = (\mu - 2\sigma, \mu + 2\sigma)$$

11. Mean/ st. Deviation of Binomial Distribution/ Repeated Events

$$\mu = np$$

$$\sigma^2 = np(1 - p)$$

Same formulas above for 68%, 96% CI.

PROBABILITY

Topic 1---MEAN

The mean is obtained by adding a set of values and dividing by the number of values.

For example, if your three test grades are 80, 90, and 100, then the mean is $(80+90+100)/3=90$.

We use the greek letter sigma to represent sum.

The formula for the mean becomes: $\text{mean} = \frac{\sum x}{n}$

We use \bar{x} to define the mean of a set of numbers.

Topic 2 ---MEDIAN&MODE

MEDIAN

The median is the middle value, when a set of data is arranged in order of magnitude. For example, if your test grades are 90, 10, 100, your median test grade is 90. It is the middle number, when you arrange your scores in order: 10, 90, 100. The mean would be $200/3=67$

Which average would you like?

If there are two middle scores, take the mean of them to compute the median.

MODE

The poor lonely mode is the number that is most frequent. We rely on the mean and to some extent the median in Statistics.

HW

1. Find the mean and median of the following salaries:

40, 20, 60, 100, 80.

2. Find the average arrival time (mean) of customers in bank, if you let:

8am=0, 10am=2, noon=4, and 4pm=8.

8am, 10am, noon, 3pm.

3. Find the mean and median GPA:

3, 2.2, 3.6, 1.4.

VARIATION---Topic 3 and 4

VARIANCE= Topic 3

The rang of a set is the largest minus the smallest. For example, if your salary ranges from 50k to 70k to 90k over three years. The rang=90k-50k=40k.

A much more useful definition of variation is the variance. We will use the short-cut definition of variance.

N= the number of values

$\sum x$ = sum of values

$\sum x^2$ = sum of the sequences of the values

$$\text{Variance} = \sigma^2 = \frac{n \sum x^2 - (\sum x)^2}{n(n-1)}$$

The standard deviation is the square root of the variance.

For example take the numbers 1, 3, and 14.

N=3

X=18

$x^2=206$

Substitute in the formula for variance:

$$\text{Variance} = \sigma^2 = s^2 = \frac{3(206) - 18^2}{3(3-1)} = 49$$

Then the standard deviation = $\sqrt{49} = 7$

The coefficient of variance = standard deviation *100%/mean = $7*100\%/6=117\%$

$$\text{The formula definition of variance} = \sigma^2 = \frac{\sum (x - \text{mean})^2}{n}$$

HW

1. Find the variance, standard deviation, coefficient of variance, and range of the data from questions 1 and 2 from the previous homework.

Topic 5---CORRELATION

Correlation is the most important topic in Statistics. We will cover it twice.

A correlation shows a relationship between 2 variables. The correlation ranges from -1 (Highest inverse correlation) to +1 (Highest direct correlation).

Inverse means that if x goes up, y goes down. Direct means that if x goes up, y goes up.

Formula

$$R = \text{correlation} = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

Coefficient of Determination= R^2 = the amount of variation in Y that is explained by the linear relationship between x and y.

HW

1. X Advanced Math Courses	0	4	8
Y Salary	30	50	100

Find the correlation between math courses and salary.

2. (x,y)= (Homework Time per week, GPA)

(3, 2.5) (5, 2) (10, 4)

Find the correlation between absences and Math Grade.

3. X-absences	Y- Math Grades
6	70
1	90
4	80

Find the correlation between absences and Math Grade.

PROBABILITY BASICS

1. Probability of an event = $p(x) = f/x$. F is the number of ways x can happen. N is the total number of outcomes.

Ex. Find the probability of flipping a coin and getting a head.

Ex. Pick a card from a 52 card deck. Find the probability of an Ace. Find the probability of a spade.

2. Probability ranges from 0, impossible, to 1, certainty.

Ex. Find the probability of jumping to moon.

Ex. Find the probability of getting either a head or a tail.

Rules

1. $P(\text{not } x) = 1 - P(x)$

Ex. Find the probability of not getting a head.

Ex. Find the probability of not selecting an Ace from a 52 card deck.

2. If A and B are any two events, $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$.

$A \cap B$ is the overlap. You have counted this probability twice.

Ex. Find the probability of selecting an Ace or a King in a single pick.

Ex. Find the probability of flipping either a head or a tail in one flip.

Ex. Find the probability of selecting an Ace or a heart in one pick.

Ex. Find the probability of selecting an Ace or a spade or a club in one pick.

Ex. Find the probability of selecting a queen or a king or a heart in one pick.

HW

Write a 1-2 page paper on your perspective on CERTAINTY.

2 points extra credit.

You must visit and discuss during office hours for credits.

Topic 6---One event- two or more possible outcomes---addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = P(A \text{ and } B)$$

$P(A \cap B)$ = over lap = the probability you counted twice.

Ex. If you flip a coin, $P(\text{head or tail}) = 1/2 + 1/2 = 1$

Ex. If you pick a card from a 52 card standard deck, $P(\text{Ace or King}) = 4/52 + 4/52$
 $= 8/52 = 2/13$

Ex. If you find $P(\text{Ace or a Heart}) = P(\text{Ace}) + P(\text{Heart}) - P(\text{Overlap- the Ace of Hearts})$
 $= 4/52 + 13/52 - 1/52 = 16/52 = 4/13.$

HW

1. In a 52 card deck (one pick), find:

$P(\text{Spade or Heart})$, $P(\text{not spade})$, $P(\text{card higher than 10})$.

2. For the same deck, find:

$P(\text{Ace or Heart})$, $P(\text{Ace or Spade or Club})$, $P(\text{Ace or Deuce or Diamond})$.

3.	Male	Female
High Math	6	14
Low Math	10	20

From the above data,

(a). Find $P(\text{High Math Male or High Math Female})$.

(b). $P(\text{Low Math Male or Male})$.

(c). $P(\text{High Math Male or Low Math Female})$.

Topic 7

Two or more successive events---MULTIPLICATION RULE

If A and B are any two events $P(A \text{ and } B) = P(A) * P(B|A)$

$(B|A)$ means the probability of B after A happens.

Ex. Find the probability of two consecutive heads.

Ex. Find the probability of five consecutive births being boys.

Ex. Find the probability of picking two consecutive Aces:

- a. With replacing the first Ace.
- b. Without replacing the first Ace.

Ex. Find the probability of selecting two consecutive spades:

- a. With replacing the first spade.
- b. Without replacing the first spade.

Ex. Find the probability of selecting three consecutive kings:

- a. With replacing each king.
- b. Without replacing each king.

Ex. Male 22

Female 18

In our class of 40 students, find the probability of selecting two consecutive males.

You cannot pick the same person twice.

HW

1. In two consecutive picks from a standard 52 card deck, find: $P(\text{drawing 2 Aces})$

- a. With replacement.
- b. Without replacement.

2. Find $P(\text{having 6 consecutive boys})$

3.	A in Math	No A
Male	6	12
Female	12	10

Find $P(\text{selecting 2 consecutive females with an A in Math})$

- a. With replacement.
- b. Without replacement.
- c. THOUGHT QUESTION-Who does better in Math?

Topic 8---counting rules

Suppose there are several actions where the first has x possibilities, the second b possibilities, the third c possibilities, and so on. To find the total number of possibilities for all the actions, multiply a, b, and c.

HW

1. How many outfits can you wear if you have ten shirts, five pants, and four sweaters?
2. How many 7 digit phone numbers are possible? You cannot use 0 for the first digit.
3. How many pitcher- catcher pairs can you have on a baseball team with 5 pitchers and 4 catchers?
4. How many ways can you go to Canada if there are 5 ways to get to Albany and 6 ways to get from Albany to Canada?

2. FACTORIAL

$$K! = k(k-1)(k-2) \dots * 1$$

$$5! = 5*4*3*2*1 = 120$$

$$6! = 6*5*4*3*2*1 = 726$$

$$\text{Ex. } 4! =, 7!, 10!, 1!, 0! =.$$

PERMUTATIONS

A permutation of r objects from a set of m objects is any set with r objects where order is important.

Ex. List all the 2 letter permutations from (a,b,c). How many 2 letter permutations are possible from a set of 3 letters.

$$\text{Permutation Formula } mPr = \frac{m!}{(m-r)!} \quad m = \text{total number of objects}$$

$$r = \text{number of objects selected}$$

HW

1. How many lottery tickets of 2 numbers can be selected from a total of 10 numbers,

if order matters?

2. How many exacta race tickets are possible from a field of 8 horses? Exacta means the winner and second place horse.
3. How many ways can you put 5 books on a shelf?
4. How many pairs of president/ vice president can be formed from our 36 student class?

COMBINATIONS

A combination is identical to a permutation, but order does not matter. A combination ab is the same as ba . For a permutation ab is different from ba .

Combination formula: $mCr = \frac{m!}{r!(m-r)!}$

HW

1. How many pairs of student representatives can be made from our 36 student class?
2. How many 2 card poker hands can be made from a poker deck?
3. How many of 3 students are from a 20 student class?

BINOMIAL PROBABILITY FORMULA---Formula 9

Assumptions

1. You are repeating n identical trials.
2. Two possible outcomes-success or failure.
3. Success probability stays the same each trial.
4. Trials are independent.

Formula $P(x) = \text{probability of } x \text{ successes in } n \text{ trials} = {}^nC_x p^x (1-p)^{n-x}$

N= total number of trials

X= number of successes

P= probability of success

nC_x = combination of n items taken x at a time

HW

1. Find the probability of exactly 2 heads in three coin flips.
2. If you select a card from a deck 3 times and replace it each time find the probability of selecting exactly one Ace. At least two Aces.
3. It rains in the Adirondacks 40% of the time. In a 4 day weekend find probability of no day without rain.

Thought question.

I researched ESP years ago. One of my students rolled a die 54 times and got 29 correct rolls. What would you conclude? Use binomial probability.

Formula 10---MEAN OF A DISCRETE RANDOM VARIABLE

Definition: a random variable is a numerical variable whose value depends on chance.

We could use frequencies to approximate probabilities, if they are not easily known.

Discrete means can be counted or put in a list.

Ex. $P(\text{head}) = 1/2$ is the probability of a head or a tail. X can take on the value head or tail.

$$P(\text{Ace in one pick}) = 4/52 = 1/13$$

$$\text{MEAN OF A DISCRETE RANDOM VARIABLE} = \sum xP(x)$$

$$\text{Mean of this gamble} = \$100 * 1/2 + (-\$200) * 1/2 = -\$50$$

HW

1. If you pick an Ace you win \$40. If you do not, you lose \$5. Find the mean of this random variable.
2. Heads I win \$100. Tails you win \$10. Find your expected value.
3. A stock has a 40% chance of earning \$10,000 and 60% chance of losing \$5000. Find the mean.

Formula 11---MEAN AND STANDARD DEVIATION OF BINOMIAL DISTRIBUTION

We have learned about binomial distribution.

We start with total trials n , number of success x , and probability of success p .

The mean $u = np$

$$\sigma^2 = \text{variance} = np(1-p)$$

$$\sigma = \text{standard deviation} = \sqrt{np(1-p)}$$

An approximate 95% confidence interval to estimate the total number of successes in a binomial trial is $(u - 2\sigma, u + 2\sigma)$

An approximate 99% confidence interval to estimate the total number of successes in a binomial trial is $(u - 1\sigma, u + 1\sigma)$

HW

Find a 68% confidence interval and 95% confidence interval for:

1. The total number of heads of you flip a coin for 1000 times.
2. The total number of spades if you select a card from a standard deck 20 times with replacement.
3. The total number of a job offers you may receive if you mail out 500 resumes with a 1% chance of job interview and a 20% chance of a job interview translating to a job offer. Hint: find the probability that a resume results in a job offer.

TI 83

\bar{x} , σ for (2, 3, 4)

1. Set up List 1 for (2, 3, 4)

- ① 2nd OFF, Calculator on.
- ② 2nd (
- ③ 2, 3, 4
- ④ 2nd)
- ⑤ Press sto
- ⑥ 2nd 1 this puts (2, 3, 4)→L1 List 1
- ⑦ Enter
- ⑧ 2nd List ◀
- ⑨ ▼ go down to the mean
- ⑩ Enter
- 11 Press 2nd 1 this is List 1
- 12 Enter answer is 3
- 13 2nd List ◀
- 14 ▼ go to standard deviation
- 15 Enter
- 16 2nd 1
- 17 Enter answer is 1
- 18 Try now yourself.

TI 83 Correlation

1. Enter List 1 (1, 2 0)
2. Enter List 2 (2, 3 ,1)
3. Press stat▶
4. ▼ go down to LinReg(ax+b)
5. Enter
6. Enter

Answer

$$Y = ax + b$$

$$A = 1$$

$$B = 1$$

$$r^2 = 1$$

$$r = 1$$

PROBABILITY-HOMEWORK & ANSWERS

TOPIC 1/2 mean/median

$$1. \text{mean} = 300/5 = 60$$

$$\text{median} = \text{middle number} = 60$$

$$2. \bar{x} = \text{mean} = 3.25$$

$$\text{median} = 3$$

$$3. \text{mean} = 2.55$$

$$\text{median} = 2.6$$

TOPIC 3/4 variance/st. Deviation

$$1. X = 40, 20, 60, 100, 80$$

$$\sigma^2 = \text{variance} = 1000$$

$$\sigma = \text{st. Deviation} = \sqrt{1000} = 32$$

$$\text{Range} = 80$$

$$\text{Coefficient of variation} = 31.6/60 * 100\% = 53\%$$

$$\text{For } x = 0, 2, 4, 7$$

$$\sigma^2 = 9$$

$$\sigma = 3$$

$$\text{Range} = 7$$

$$\text{Coefficient of variation} = 92\%$$

TOPIC 5 Correlation

$$1. R = 1 \quad \text{highest correlation}$$

$$2. R = 0.87$$

$$3. R = -1 \quad \text{highest inverse}$$

TOPIC 6 P(A or B)

$$1. P(\text{Spade or Heart}) = 26/52 = 1/2$$

$$P(\text{not Spade}) = 1 - 1/4 = 3/4$$

$$P(\text{card higher than 10}) = 16/52 = 4/13$$

$$2. P(\text{Ace or Heart}) = 4/13$$

$$P(\text{Club or Spade or Ace}) = 13/52 + 13/52 + 4/52 - 2/52 = 7/13$$

$$P(\text{Ace or Deuce or Diamond}) = 19/52$$

$$3. P(\text{high male or high female}) = 2/5$$

$$16/50 = 8/25$$

$$13/25$$

TOPIC 7 P(A and B)

$$1. a) \frac{1}{13} * \frac{1}{13} = \frac{1}{169}$$

$$b) \frac{4}{52} * \frac{3}{51} = \frac{1}{221}$$

$$c) \frac{13}{52} * \frac{12}{51} = \frac{1}{17}$$

$$2. \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$3. a) \frac{12}{40} * \frac{11}{39} = \frac{11}{130}$$

$$b) \frac{12}{40} * \frac{12}{40} * \frac{9}{100}$$

TOPIC 8 Counting

$$1. 10 * 5 * 4 = 200$$

$$2. 9 * 10^6 = 9,000,000$$

$$3. 5 * 4 = 20$$

$$4. 5 * 6 = 30$$

Permutations

$$1. 10P_2 = \frac{10!}{(10-2)!} = 10 * 9 = 90$$

$$2. 8P_2 = \frac{8!}{6!} = 56$$

$$3. 5P_5 = \frac{5!}{(5-5)!} = \frac{120}{1} = 120$$

$$4. 36P_2 = 1260$$

Combinations

$$1. 36C_2 = \frac{36!}{(36-2)!2!} = 630$$

$$2. 52C_2 = \frac{52!}{50!2!} = 1326$$

$$3. 20C_3 = \frac{20!}{(20-3)!3!} = 1140$$

TOPIC 9 Binomial Probability

$$1. P(x=2) = 3C_2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right)^{3-2} = \frac{3}{8}$$

$$2. P(x=1) = 3C_1 \left(\frac{1}{13}\right)^1 \left(1 - \frac{1}{13}\right)^{3-1} = \frac{432}{2197}$$

$$P(\text{at least 2 Aces}) = P(X=2) + P(X=3)$$

$$= 3C_2 \left(\frac{1}{13}\right)^2 \left(1 - \frac{1}{13}\right)^{3-2} + 3C_3 \left(\frac{1}{13}\right)^{3-3}$$

$$= \frac{36}{2197} + \frac{1}{2197}$$

$$= \frac{37}{2197}$$

$$3. P(x=4) = 4(4(0.4)^4 (1-0.4)^{4-4}) = \frac{4!}{4!0!} (0.4)^4 (1) = 0.0256$$

TOPIC 10 mean of a R.V.

$$1. E(x) = \frac{1}{2}(40) + \frac{1}{2}(-5) = 17.5$$

2. -45

3. 1000

TOPIC 11 mean/s.d. Binomial Distribution

$$1. \sigma^2 = np(1-p) = 1000\left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right) = 250$$

$$\sigma = \sqrt{250} = 16$$

$$\mu = np = 500$$

$$68\%CI = (500-16, 500+16) = (484, 516)$$

$$95\%CI = (500-32, 500+32) = (468, 532)$$

$$2. \mu = \frac{1}{4}(20) = 5, \sigma^2 = 20\left(\frac{1}{4}\right)\left(1-\frac{1}{4}\right) = 3.75$$

$$\sigma = \sqrt{3.75} = 2$$

$$68\%CI = (5-2, 5+2) = (3, 7)$$

$$95\%CI = (5-4, 5+4) = (1, 9)$$

$$3. P = 0.01(0.2) = 0.02, \mu = 0.002(500) = 1$$

$$\sigma^2 = 1, \sigma = 1$$

$$68\%CI = (0, 2)$$

$$95\%CI = (-1, 3)$$

*-1 is absurd but correct

Optional additional Hw

10th Ed. Triola-- Elementary Statistics

Topic 1 and 2--p 87, 5-15

Topic 3 and 4--p106-107, 11-20

Topic 5--p534, 9, 10, 13, 15

Topic 6 -- $P(A \text{ or } B)$ --p156, 7 13, 16, 18,19

Topic 7-- $P(A \text{ and } B)$ --p165, 7, 8, 10, 12, 17, 18

Topic 8--Counting Permutation/ Combination, p186, 3, 4, 5, 9, 10, 13, 14, 18,19 24,
27, 30

Topic 9--Binomial, p222, 17, 21, 31, 32, 33

Topic 10--Expected Value, p213, 17, 18, 19, 20

Topic 11--Mean/ Binomial, p228, 11, 13. Find 68%, 95% CI.

WALL STREET JOURNAL STUDY (2009) BEST JOBS IN AMERICA- Mathematician, Actuary, Statistician (In Dr. Loase's opinion mathematics teacher/professor). Take Calculus 1 and 2 until you earn at least an A or B. Then train for one of these top jobs, taking advanced mathematics.

Mathematics area of Study leads you to them as follows:

1. Calculus 1, 2, and 3 together with Calculus Based Probability and Statistics is the minimum coursework to prepare you to take Actuarial Test 1 – the entry level exam. Consult the website of the Society of Actuaries for additional information and sample test questions. My personal recommendation is that you also take Linear Algebra, Discrete Mathematics, Mathematical Computer Programming, and join the Concordia College Math Team as early as possible.

Actuary James Rowe advises “ We encourage anyone interested in exploring an actuarial career to visit the website BeAnActuary.com. It is filled with information on the exam process and also gives practical insight into what an actuary does and what it takes to become an actuary. “

2. To train for Statistician, enter a Masters Degree program at Columbia University or SUNY-Stony Brook in Statistics. The Director of Graduate Statistics at Columbia recommended that you take (in addition to #1 above) Linear Algebra, Intro. To Mathematical Computer Programming , and learn applications of advanced mathematics to real-world problems. Applications to the real world are the training that we provide to the Concordia College Math Team.
3. Either route prepares you for a Masters degree in Mathematics Education. There is a huge need for secondary mathematics teachers. Go where the jobs are. Join the Concordia College-NY Math Team as early as possible. The training is real-world, vital applications of Mathematical Modeling.

Several Concordia College-NY graduates are thriving today in their careers as Statisticians, Actuaries, and high school mathematics teachers. Join them. If you wish guidance from a practicing actuary, statistician, or mathematics teacher, contact Dr. Loase.

The Concordia College Mathematics team regularly performs at or above the levels of MIT, Princeton, Cornell, Yale, and RPI in the International Contest in Mathematical Modeling. You need an A or B in Calculus 1 and 2 to earn the \$1000 Math Team scholarship. Put mathematics in your future.

MATH IS FUN, MATH IS POWER-GET INVOLVED.

Dr. John F. Loase Professor of Mathematics-Concordia College-NY

Doing the Math to Find the Good Jobs

Mathematicians Land Top Spot in New Ranking of Best and Worst Occupations in the U.S.

By SARAH E. NEEDLEMAN

Nineteen years ago, Jennifer Courter set out on a career path that has since provided her with a steady stream of lucrative, low-stress jobs. Now, her occupation — mathematician — has landed at the top spot on a new study ranking the best and worst jobs in the U.S.

"It's a lot more than just some boring subject that everybody has to take in school," says Ms. Courter, a research mathematician at mental images Inc., a maker of 3D-visualization software in San Francisco. "It's the science of problem-solving."

The study, released Tuesday from CareerCast.com, a new job site, evaluates 200 professions to determine the best and worst according to five criteria inherent to every job: environment, income, employment outlook, physical demands and stress. (CareerCast.com is published by Adicio Inc., in which Wall Street Journal owner News Corp. holds a minority stake.)

The findings were compiled by Les Krantz, author of "Jobs Rated Almanac," and are based on data from the U. S. Bureau of Labor Statistics and the Census Bureau, as well as studies from trade associations and Mr. Krantz's own expertise.

According to the study, mathematicians fared best in part because they typically work in favorable conditions — indoors and in places free of toxic fumes or noise — unlike those toward the bottom of the list like sewage-plant operator, painter and bricklayer. They also aren't expected to do any heavy lifting, crawling or crouching — attributes associated with occupations such as firefighter, auto mechanic and plumber.

The study also considers pay, which was determined by measuring each job's median income and growth potential. Mathematicians' annual income was pegged at \$94,160, but Ms. Courter, 38, says her salary exceeds



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COLLEGE PROBABILITY AND STATISTICS

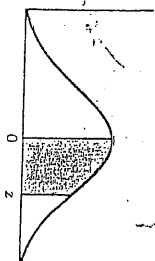
PART 2 Statistics Homework

Z Scores, Confidence Intervals, Sample Size, One Sample
Hypothesis Tests Difference of Means

Pages 1-10

1

The standard normal (z) distribution



100%

z	00	01	02	03	04	05	06	07	08	09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

2

Student's t distribution



+ Distribution - table

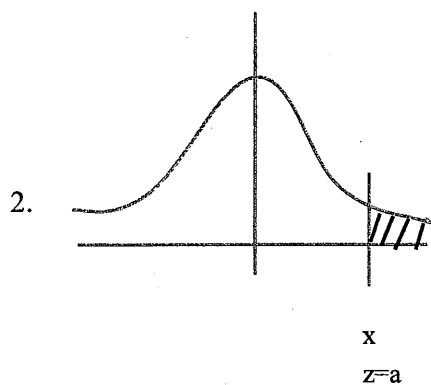
	$t_{0.99}$	$t_{0.95}$	$t_{0.90}$	$t_{0.85}$	$t_{0.80}$	$t_{0.75}$	$t_{0.70}$	$t_{0.65}$	$t_{0.60}$	$t_{0.55}$	$t_{0.50}$	$t_{0.45}$	$t_{0.40}$	$t_{0.35}$	$t_{0.30}$	$t_{0.25}$	$t_{0.20}$	$t_{0.15}$	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.08	6.31	12.71	31.82	63.66																		
2	1.89	2.92	4.30	6.96	9.92																		
3	1.64	2.35	3.18	4.54	5.84																		
4	1.53	2.13	2.78	3.75	4.60																		
5	1.48	2.02	2.57	3.36	4.03																		
6	1.44	1.94	2.45	3.14	3.71																		
7	1.42	1.89	2.36	3.00	3.50																		
8	1.40	1.86	2.31	2.90	3.36																		
9	1.38	1.83	2.26	2.82	3.25																		
10	1.37	1.81	2.23	2.76	3.17																		
11	1.36	1.80	2.20	2.72	3.11																		
12	1.36	1.78	2.18	2.68	3.05																		
13	1.35	1.77	2.16	2.65	3.01																		
14	1.35	1.76	2.14	2.62	2.98																		
15	1.34	1.75	2.13	2.60	2.95																		
16	1.34	1.75	2.12	2.58	2.92																		
17	1.33	1.74	2.11	2.57	2.90																		
18	1.33	1.73	2.10	2.55	2.88																		
19	1.33	1.73	2.09	2.54	2.86																		
20	1.33	1.72	2.09	2.53	2.85																		
21	1.32	1.72	2.08	2.52	2.83																		
22	1.32	1.72	2.07	2.51	2.82																		
23	1.32	1.71	2.07	2.50	2.81																		
24	1.32	1.71	2.06	2.49	2.80																		
25	1.32	1.71	2.06	2.49	2.79																		
26	1.32	1.71	2.06	2.48	2.78																		
27	1.31	1.70	2.05	2.47	2.77																		
28	1.31	1.70	2.05	2.47	2.76																		
29	1.31	1.70	2.05	2.46	2.76																		
30	1.28	1.64	1.96	2.33	2.58																		

Adapted from Donald R. Brown

Z Score and Corresponding Probabilities

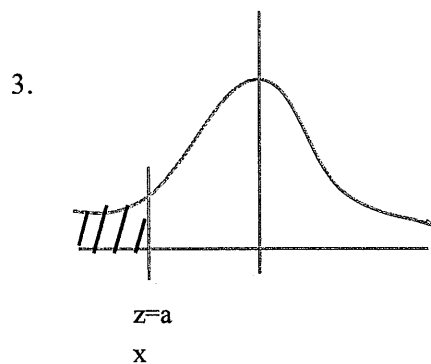
1. $z = \frac{x - \mu}{\sigma}$

Z score tells you how
many standard deviations
the score x is away from
the mean μ



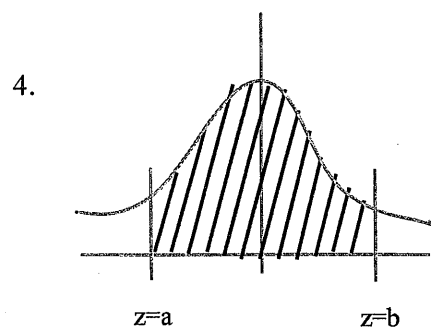
To find $P(z > a)$

1. Find $P(z < a)$ from table
2. Answer is $1 - P(z < a)$



To find $P(z < a)$

1. Find $P(z < a)$ from table



To find $P(a < z < b)$

Subtract two probabilities

$$P(z < b) - P(z < a)$$

Z score Homework

$$z = \frac{x - \mu}{\sigma}$$

For IQ, $\mu = 100$, $\sigma = 10$

For SAT, $\mu = 500$, $\sigma = 100$

1. Find probability of a person having an IQ (a) greater than 100, (b) less than 110
2. Find percentage of students scoring (a) higher than 600 on the math SAT, (b) lower than 600 on the math SAT
3. (a) Find $P(\text{IQ} < 80)$
(b) Find $P(\text{IQ} > 80)$
4. Find $P(90 < \text{IQ} < 120)$
5. Find $P(350 < \text{SAT} < 700)$

Answers:

1. (a) 16% (b) 84%
2. (a) 16% (b) 84%
3. (a) 2% (b) 98%
4. 82%
5. 91%

Confidence Interval Homework

1. 100 Concordia graduates have average 40k salary, $\sigma = 3k$. Find 95% and 99% confidence intervals for Concordia graduates' salary.
2. The mean IQ for over 30 student class is 110, $\sigma = 16$. Find 95% and 99% confidence intervals for the college mean IQ.
3. 50 customers in your store, average \$24 in purchase, $\sigma = \$10$. Find 95% and 99% confidence intervals for average customer purchase.
4. 1000 college graduates of SUNY graduate with a mean GPA of 3.0, $\sigma = 1$. Find 95% and 99% confidence interval for SUNY graduates' mean GPA.
5. Complete question #1 if sample size is 10.
6. Complete question #2 if sample size is 4.
7. Complete question #3 if sample size is 5.
8. Complete question #4 if sample size is 25.

Answers:

- | | |
|-------------------------------|---------------------------|
| 3. (21, 27), (20, 28) | 2. (104, 116), (102, 118) |
| 7. (12, 36), (3, 45) | 6. (85, 135), (63, 157) |
| 4. (2.94, 3.10), (2.92, 3.08) | 1. (39, 41), (39, 41) |
| 8. (2.59, 3.41), (2.44, 3.56) | 5. (38, 42), (37, 43) |

SAMPLE SIZE

- To determine n that will ensure a certain level of confidence and accuracy.

To determine the size of a sample, start with: 95% or 99% confidence interval for μ

$$95\% \text{ CI for } \mu = \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\text{Error} = e = (1.96) \frac{\sigma}{\sqrt{n}}, \text{ solve for } n.$$

Next, find corresponding n for a 99% C.I.

$$n = \left[\frac{(2.57)\sigma}{e} \right]^2$$

Ex 1. Find the sample size needed to estimate the mean age of marriage for men if we want to be within $\frac{1}{2}$ year of the true age with 95% confidence, let $\sigma=4$.

Ex 2. We want to estimate the mean IQ of Lawyer. How many Lawyers are needed for 95% confidence, the sample mean is within 2 IQ points of the population mean. $\sigma=12$.

Ex 3. Find the number of quarters needed to estimate sample mean within 0.025g, if $\sigma= 0.068\text{g}$, and 99% confidence.

$$1. N=246$$

$$2. N=139$$

$$3. 49$$

SAMPLE SIZE HOMEWORK

1. How many women need to be sampled to be 95% confident of estimating the mean marriage age within 1/2 year if the standard deviation is 4 years.
2. Determine sample size to be 99% confident of estimating newborn weight within 6 oz. if the variance is 100 ounces.
3. Complete #1 with 99% confidence.
4. Complete #2 with 99% confidence.
5. How many Concordia students should be sampled to attain 99% confidence of the mean college GPA within 0.2 if the mean is 3.0 with standard deviation 1.
6. Complete problem 5 with 95% confidence.

Answers

Do not look until you completed the HW.

1. 246
2. 19
3. 423
4. 11
5. 166
6. 97

Practical problems follow on confidence interval.

Practical Problems II

A. Review Test 1 and 10 topics for Test 1

These are 5 new topics.

1. Of 870 juniors, 20% were disgunlified . Find a 95% confidence interval for the number disgunlified.
 2. For an IQ test, $\mu=100$, $\sigma=20$. Find (a) $P(IQ>140)$, (b) $P(80<IQ)$.
 3. Find a 95% CI for the mean salary of doctors, if a smple of 50 doctors had a mean of 110k, $\sigma=20$.
 4. Change the sample size in above example to 16. Find a 99% confidence interval for the population mean salary. ($n=16$)
 5. Find the sample size needed to be 95% confidence of estimating salary within 10k, if $\sigma=50$.
 6. Find $P(IQ>148)$ IF $\sigma=12$, $\mu=100$.
-
1. (672, 720)
 2. (a) 2%, (b)68%
 3. (104, 116)
 4. (102, 118)
 5. 97
 6. 0

Practice Problems Test 2

1. IQ is normally distributed with $\mu=100$, $\sigma=10$

Find (a). $P(IQ>120)$

(b). $P(90<IQ<120)$

(c). $P(IQ>130)$

2. You get a hit 30% of the time. Find a 95% confidence interval for number of hits in 1000 times.

3. Average GPA is 2.8, $\sigma=1.2$. For a sample of 36, find 95% and 99% confidence intervals for population mean GPA.

4. Find the sample size needed to be (a) 95%, (b) 99% confident of estimating income within 2k if $\sigma=40k$.

5. Asian women live on average to be 88, $\sigma=12$. Is this above the U.S. average of 82. Sample size=100.

6. Change sample size to 20. What do you conclude? $\alpha=0.05$, $\alpha=0.01$

Answers

6. $T=2.24$, $t_{0.01,19} = 2.54$ $\alpha=0.01$, Accept H_0 , $\bar{x} = \mu$

$\alpha=0.05$, $t=1.73$, $\bar{x} > \mu$

5. $Z=5$, accept H_a , $\bar{x} > \mu$

4. 95%: $n=1537$, 99%: $n=264$

3. $95\%CI = 2.8 \pm 1.96\left(\frac{1.2}{6}\right) = 2.8 \pm 0.04 = (2.4, 3.2)$

$$99\%CI = 2.8 \pm 0.5 = (2.3, 3.3)$$

$$2.300 \pm 28 = (272, 328)$$

1.(a) 2%

$$(b) 34\% + 48\% = 82\%$$

$$(c) 0.0013 = 0.13\%$$

COLLEGE PROBABILITY AND STATISTICS

PART 3 Statistics Homework

Two Sample Hypothesis Tests Difference of Means, One and Two Sample Difference of Proportions, Regression , Correlation, Chi- Square Goodness of Fit Test.

Pages 1 - 19

HOMEWORK

HYPOTHESIS TESTING- 2 SAMPLE DIFFERENCE OF MENS

1. 306 women score a mean in Potential for Sigfluence of 50, st dev=9

236 men average 45, st dev=45.

Do women score higher Type 1 alpha error=5%, 1%?

2. 100 Concordia grads have average salary of \$80k, st dev=30;

150 College X grads have average salary of \$470k, st dev=20k.

Is the mean for Concordia higher? Alpha=5%.

3. 10 men averaged 82 on the Statistics final with se dev of 20.

8 women averaged 84, st dev=24.

Did women score significantly higher? Alpha=1%.

4. 8 men averaged 6 hours study each night with st dev of 2.

5 women averaged 5 hours of study , st dev=1. Did men study more? Alpha=5%

5. 1000 Westchester students scored a mean of 510 on the Math SAT, 1200 Westport

students averaged 500. St dec=100 for both groups. Did Westchester score

significantly higher? Alpha=5%

6. Change the sample size to 10 for Westchester and Westport. Did Westchester scpre

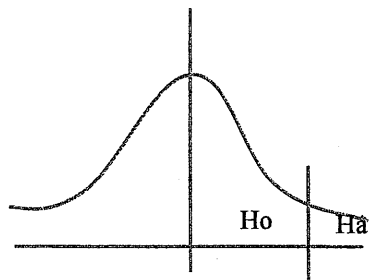
significantly higher with all else the same? Alpha=1%

Homework- Hypothesis Testing

$$1. z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{50 - 45}{\sqrt{\frac{9^2}{306} + \frac{45^2}{236}}} = \frac{5}{\sqrt{8.845}} = \frac{5}{2.97} = 1.68$$

$$H_0: \bar{x}_1 = \bar{x}_2$$

$$H_a: \bar{x}_1 > \bar{x}_2$$

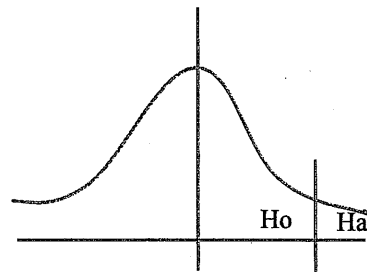


$$Z=1.64$$

Accept H_a , $1.68 > 1.64$

$$\bar{x}_1 > \bar{x}_2$$

$$\alpha = 5\%$$



$$Z=2.33$$

Accept H_0 , $1.68 < 2.33$

$$\bar{x}_1 = \bar{x}_2$$

$$\alpha = 1\%$$

2. Sample formula

$$\bar{x}_1 = 80, \sigma_1 = 30,$$

$$\bar{x}_2 = 47, \sigma_2 = 20, n_2 = 150$$

$$Z = 9.66 > 1.64$$

Accept H_a , $\bar{x}_1 > \bar{x}_2$, $\alpha = 5\%$

$$3. t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{Sp^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad Sp^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$n_1 = 10, \bar{x}_1 = 82, \sigma_1 = 20$$

$$n_2 = 8, \bar{x}_2 = 84, \sigma_2 = 24$$

$$Sp^2 = \frac{(10-1)20^2 - (8-1)24^2}{10+8-2} = 477 \quad \text{put large number first}$$

$$t = \frac{84-82}{\sqrt{477\left(\frac{1}{10} + \frac{1}{8}\right)}} = \frac{2}{10.4} = 0.19 < 2.58 \quad t_{0.01,16} df = 2.58$$

Accept Ho: $\bar{x}_1 = \bar{x}_2$, $\alpha=0.01$

$$4. n_1 = 8, \bar{x}_1 = 6, \sigma_1 = 2$$

$$n_2 = 5, \bar{x}_2 = 5, \sigma_2 = 1$$

$$Sp^2 = \frac{(n_1-1)S_1^2 - (n_2-1)S_2^2}{n_1+n_2-2} = \frac{7 \cdot (2)^2 + 5 \cdot (1)^2}{8+5-2}$$

$$Sp^2 = 3$$

Use same formula as No. 3

$$t = \frac{6-5}{\sqrt{3 \cdot \left(\frac{1}{8} + \frac{1}{5}\right)}} = \frac{1}{0.99} = 1.01$$

$$df=8+5-2=11$$

$$t_{0.05,11df} = 1.80$$

$$1.01 < 1.80$$

Accept Ho, $\bar{x}_1 = \bar{x}_2$

$$\alpha = 0.05$$

$$5. z = \frac{510-500}{\sqrt{\frac{100^2}{1000} + \frac{100^2}{1200}}} = \frac{10}{\sqrt{18.33}} = 2.34$$

$$2.34 > 1.64, \text{ Accept Ha, } \bar{x}_1 > \bar{x}_2$$

$$\alpha = 0.05$$

$$6. Sp^2 = \frac{(10-1) \cdot 100^2 + 9 \cdot (100)^2}{10+10-2} = 100^2$$

$$t = \frac{510 - 500}{\sqrt{100^2 \cdot \left(\frac{1}{10} + \frac{1}{10}\right)}} = \frac{10}{44.7} = 0.22$$

$$df = 10+10-2=18$$

$$t_{0.01, 18df} = 2.55$$

$$0.22 < 2.55, \text{ Accept } H_0, \quad \overline{x}_1 = \overline{x}_2$$

$$\alpha = 0.01$$

HOMEWORK

HYPOTHESIS TESTING---One and Two SAMPLE DIFFERENCE OF PROPORTIONS

1. 306 women smoke out of 1000

236 men smoke out of 600.

Do men smoke more? α error= 5%

2. 100 out of 200 Concordia grads average salary of 60k.

Is this above the U.S. Average of 40%? α = 5%

3. 10 men of 82 are on probation for grades.

8 women of 100 are on probation.

Did women do better? α = 1%

4. 80 men of 120 averaged 4 hours study each night, 50 women of 100 averaged 4 hours of study.

Did men have a greater % studying 4 hours per night? α = 5%

5. 1000 of 1500 Westchester students scored above 500 on the Math SAT. Is this above the U.S. Ratio of 50%? α = 5%

Answers

1. $Z=3.57 > 2.33$ Men smoke more. $P_2 > P_1$. α = 1%, Accept H_a

2. $Z=2.89 > 1.64$ Concordia is greater. $P_1 > P_2$. α = 5%, Accept H_a

3. $Z=0.94 < 2.33$ The groups were equal. $P_1 = P_2$. α =1%, Accept H_o

4. $Z=3.89 > 1.64$ Men studied more. $P_1 > P_2$. α = 5%, Accept H_a

5. $Z=12.9 > 1.64$ Westchester scored higher. $P_1 > P_2$. α = 5%, Accept H_a

STATISTICS HOMEWORK: HYPOTHESIS TESTS PROPORTIONS

1. 202 of 400 Calculus students are male. Is this above average? $\alpha = 0.05$
2. New Jersey's cancer death was 205 of 100,000 sampled. Is this higher than the U.S. Rate of 0.00174? $\alpha = 0.05$
3. 92 out of 150 morning students eaned an A in Statistics. 110 out of 200 afternoon students earned A. Does the morning students do better? $\alpha = 0.01$
4. 30 of 40 women excel in Calculus. 40 of 80 men excel in Calculus. Do women do better in Calculus? $\alpha = 0.05$

Answers

1. $Z=0.02$ Acc Ho, $p = \bar{p}$, $\alpha = 0.05$

This is average performance.

2. $Z=2.35$ Acc Ha, $\bar{p} > p$, $\alpha = 0.05$

It is higher than average.

3. $Z=1.19$ Acc Ho, $p_1 = p_2$, $\alpha = 0.01$

They perform equally.

4. $Z=2.62$ Acc Ha, $p_1 > p_2$, $\alpha = 0.05$

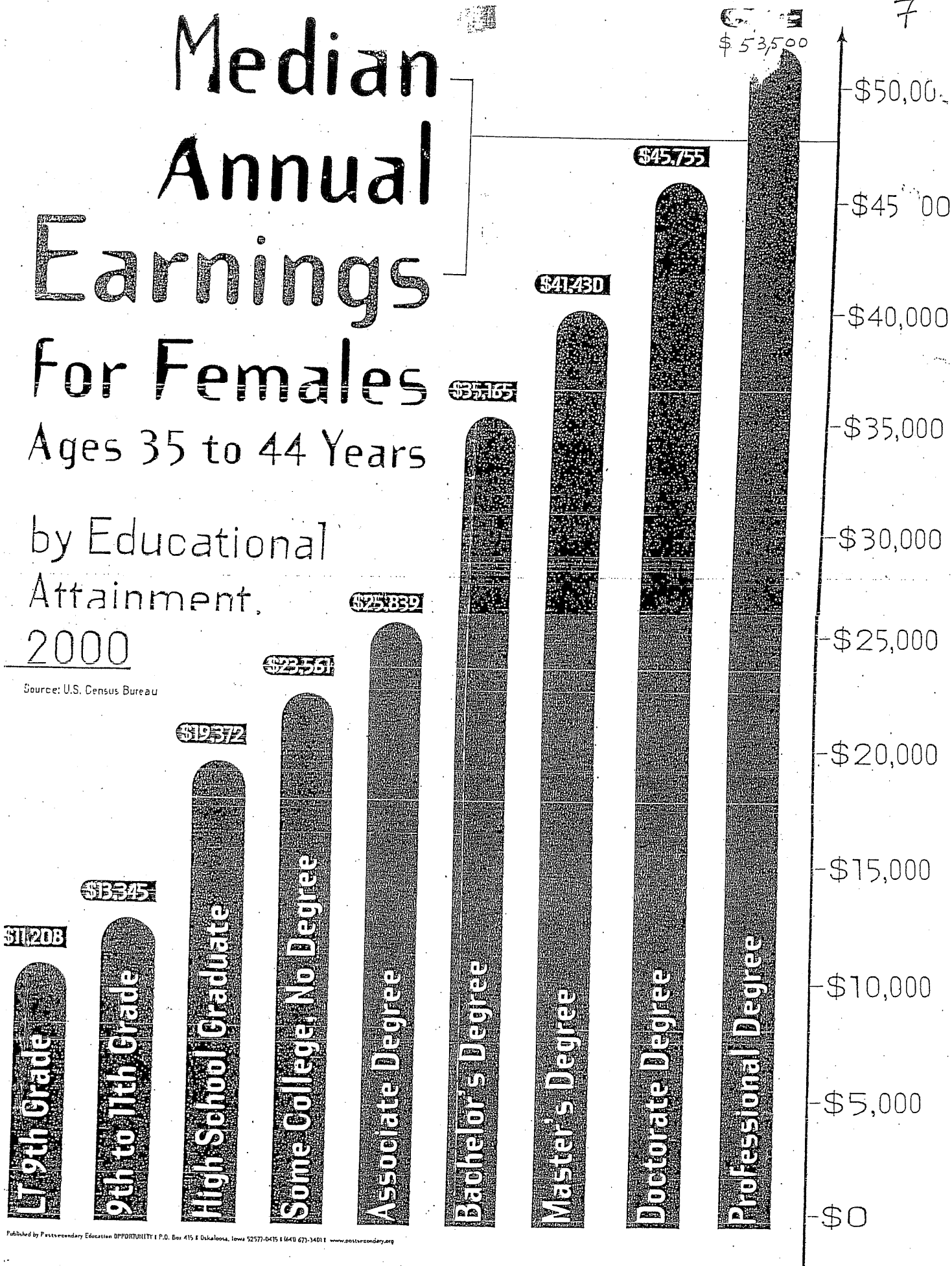
Women do better.

Median Annual Earnings for Females

Ages 35 to 44 Years

by Educational Attainment,
2000

Source: U.S. Census Bureau



NAME	PEICE	EARNING (share/year-Est)	EARNINGS GROWTH
Aetna	28	2.78	-16
EMC	17	1.06	31
Game Stop(GME)	24	2.70	16
GE	14	0.89	-20
IBM	123	10.56	9
TEVA	51	3.88	28

Nov. 5th, 2009

Let $y=a+bx$

Y =price

Use regression and correlations to analyze stock price.

What are the limitations?

- A). Small sample
- B). One day
- C). How do you know which variable is y ?
- D). Is this year in the stock market unusual?

CORRELATION REGRESSION PRACTICE

1. Find the correlation, r squared, and least squares regression line $y=ax+b$

X	Y
1	5
2	8
3	10

2. Find the correlation, r squared, and regression line $y=ax+b$

(1,2) (5,8) (4,0)

3. Find the correlation between study X (hours per week) and GPA Y. Find r squared and $y=ax+b$ also.

X	Y
2	2.0
10	3.0
20	4.0

4. Find the correlation, r squared, and regression line $y=ax+b$ for x-education in years of college and y salary.

X	Y
0	25
4	50
6	60
10	50

the last line is for professors' salaries.

Answers

1. $Y=2.5x+2.7$, r squared=0.98
 2. $Y=x$, $r=0.5$, r squared=0.25

3. $Y=0.11x+1.8$, $r=1.0$, $r^2=0.995$

4. $Y=2.6x+33.26$, $r=0.72$, $r^2=0.52$

Practice Problems CHAPTER 1,8,9,10

1. New Jersey had a death rate of 205 cancer deaths per 100,000. Is this above the national cancer death rate of 174 per 100,000? $\alpha = 0.05$

2. One group of students reported flu symptoms for 92 of 150. Of 110 students in a later class 110 of 200 report flu symptoms. At the 0.05 level, is the first group higher in flu symptoms?

3.

	n	\bar{x}	s
Team	7	2.56	0.871
Individual	8	3.04	0.621

Do individual sport participants score higher in GPA than team participants? $\alpha = 0.01$

4. 50 women have longevity of 75.5 years (mean) with 16.2 for st. Dev. 40 men evidence longevity of mean 67.9, $s=18.3$. Do women live longer? $\alpha = 0.05$

5. Our class SAT math average of 36 students was 540. The national average is 500. $\sigma = 100$. Are we above average? $\alpha = 0.01$

6.

X Apgar score	Y mortality rate
0	500
1	475
2	380
3	350
5	150

For the data, find a). r- correlation

b). $y = a + bx$ least squares regression line

Answers

1. $Z=2.35$, acc H_a , $P>0.00174$

2. $Z=1.19$, acc H_0 , $P1=P2$

3. $T=1.24$, acc H_0 , $\mu_1 = \mu_2$

4. $Z=2.06$, acc H_a , $\mu_1 > \mu_2$

5. $Z=2.4$, acc H_a , $\bar{x} > \mu$

6. $R = -0.981$

$Y=526.5-70.68x$

TEST 3 Statistics- Concordia College Dr. J. Loase

Please complete any 6 questions. Correst answer counts half; correct method counts half. Reducing is worth 1 pt.

1. Find the correlation between x and y.

X	Y
0	4
1	10
2	5

2. Calculate the least squares regression line $y=a+bx$

3. 100 women from Concordia average 90 in advanced Math. 100 men average 88.
Do women acore significantly higher in advanced math? St. Dev=20 for both groups.

4. Ten Pace University graduates starting salary average is 32k with standard deviation of 20k. Twelve Iona grdas average 30k with standard deviation of 24k. Are Pace grads earning significantly more? Alpha=5%

5. There are 28 men and 32 women in advanced methematics at Concordia NY. Is this a 50% representation? Use the chi-square test with alpha=5%.

6. 30 of our 36 student class is passing Statistics. At College of Misery, 10 of 32 are passing Statistics. Is our passing rate significantly higher? Alpha= 1%

7. 120 of 160 Concordia graduate college. Is our graduation rate significantly higher than the U.S. Rate of 40%? Alpha=5%.

Chi Square Goodness of Fit

A very useful procedure in statistics is testing whether an outcome is a good fit to our expectations. To test whether our observed frequencies are in accord with expected frequencies, we use a Chi-square goodness of fit test.

Use the formula $\chi^2 = \sum \frac{(O - E)^2}{E}$

Use a critical χ^2 table to look up $\chi^2 \alpha$ with $df = k - 1$, $k =$ number of categories

Ex. Consider a coin flip where heads is obtained 60 of 100 trials. Is the coin fair? (We expect that $p(\text{Heads}) = 0.05$)

	H	T
Obs.	60	40
Exp.	50	50

$$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(60 - 50)^2}{50} + \frac{(40 - 50)^2}{50} = 4$$

TABLE: CRIT χ^2 0.05, 1df = 3.81

$H_0: P(H) = 0.5$

$H_a: P(H), P(T) \neq 0.5$

At $\alpha = 0.05$, reject H_0 , accept H_a .

$P(H), P(T) \neq 0.05$

A Related Test is the χ^2 Test for Independence

We frequently want to study the dependence of two methods of classification. For example if we treated emphysema with two drugs A and B- we would study whether the improvement of patients was dependent on the type of drug.

Consider the below table of results of a one-year emphysema study:

Drug	Patient Condition			Total
	No Change	Improved	Greatly Improved	
A	20	35	45	100
B	15	45	50	110
Total	35	80	95	210

Using statistical notation

$$\text{All } O_{11} = 20 \quad O_{21} = 15$$

$$O_{12} = 35 \quad O_{22} = 45$$

$$O_{13} = 45 \quad O_{23} = 50$$

* L. Ottard W. Mendenhall, Understanding Statistics, PWS Kent, Boston, 1900, P. 572

The null hypothesis is that the two methods of classification are independent.

H_a = the methods are not independent

We need only to calculate expected count for each cell assuming independence. For example if drug and patient conditions are independent the probability that a person will be in cell O_{11} is:

$$\left[\frac{\text{row' total}}{n} \cdot \frac{\text{column' total}}{n} \right] = \frac{100}{210} \cdot \frac{35}{210} = 16.67$$

In thi same manner, the expected cell count= $\frac{\text{row' total} \cdot \text{column' total}}{n}$

For the remaining cells. Expected cell count.

Drug	no change	improved	greatly improved
A	16.67	38.10	45.23
B	18.33	41.90	49.77

$$\text{Use } \chi^2 = \sum \frac{(O - E)^2}{E}$$

Ho: the two classifications are independent

Ha: the two classifications are dependent

Reject Ho if: computed $\chi^2 > \text{CRIT } \chi^2$

Where CRITICAL χ^2 has $(r-1)(c-1)$ df

$r-1 = \# \text{ of rows}-1$

$c-1 = \# \text{ of columns}-1$

To complete the problem

$$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(20 - 16.67)^2}{16.67} + \frac{(35 - 38.10)^2}{38.10} + \frac{(50 - 49.77)^2}{49.77} = 1.75$$

$$\text{Df} = (3-1)(2-1) = 2$$

$$\text{CRIT } \chi^2_{0.05, 2} = 5.99$$

Therefore accept Ho the experimental and standard drugs-A and B-are equally effective.

Test for Independence

Contingency Tables

Income

	0-10k	10k-25k	25k-100k	100+k	Row Total
For	100	125	150	100	475
US	200	175	100	50	525
Column total	300	300	250	150	1000
					Grand Total

H_0 : Opinion is independent of income

H_a : Opinion is NOT independent of income

$$\text{Expected} = \frac{\text{row total} \times \text{column total}}{\text{Total } N}$$

$$e_{11} = \frac{475 \cdot 300}{1000} = 142.5 \quad e_{21} = \frac{300 \cdot 525}{1000} =$$

$$e_{12} = \frac{475 \cdot 300}{1000} = \quad e_{22} = \frac{300 \cdot 525}{1000} =$$

$$e_{13} = \frac{475 \cdot 250}{1000} = \quad e_{23} = \frac{250 \cdot 525}{1000} =$$

$$e_{14} = \frac{475 \cdot 150}{1000} = \quad e_{24} = \frac{150 \cdot 525}{1000} =$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(100 - 142.5)^2}{142.5} + \dots$$

$$Df = (r-1)(c-1) = (2-1)(4-1) = 3$$

$$\text{CRIT } \chi^2_{0.05, 3} =$$

Conclusion

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.001
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.957
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.306
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.929
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.287
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	67.565
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	80.132
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	92.906
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	105.912
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	118.135
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	129.565
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.991

From Donald B. Owen, *Handbook of Statistical Tables*, ©1962 Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of the publisher.

CHI SQUARE TEST PROBLEMS

1) Concordia has 360 female 340 male students. Is the percentage equal? $\alpha=0.05$

2) Digit 0 1 2 3 4 5 6 7 8 9

Frequency 35 0 2 1 4 24 1 4 7 2

Test whether each digit in an irrational number is obtained 10% of the time. $\alpha=0.01$

3)	Recover	do not
Group A(no medicine)	65	35
Group B(medicine)	75	25

Use the independence test to determine whether medicine influences recovery. $\alpha=0.05$

4) Complete the attached example testing the independence of income and attitude.

$\alpha=0.05$

Answers

1. $\chi^2 = \frac{200}{350} = 0.57 < 3.841$

Acc Ho: $p = \frac{1}{2}$ for both

2. $\chi^2 = 156.5$ Reject Ho, $156.5 > 6.64$

The digit do not occur 10% of the time

3. $\chi^2 = 2.38 < 3.84$, Recovery is independent of treatment

COLLEGE PROBABILITY AND STATISTICS

Part 4

Homework Mathematics of Finance

Exemplary Papers

Pages 1-8

EXPERIENTIAL STATISTICS PRACTICE

TOPIC ONE SIMPLE INTEREST

1. Find the interest on a \$3000 CD for 6 months at 3% annual interest. \$45
2. Find out what your account of \$1000 is worth at the end of one year of 1% simple interest. \$1010.
3. Determine your account value at the end of 8 months if you earn $1\frac{1}{2}\%$ on an account of \$10,000. \$10,033.33.
4. What is your simple interest rate if you earn \$2 a year on an account of \$1000. .2%.
5. Find the amount of interest you owe for one month on purchases of \$2000 on a credit card with 18% interest. \$30.

TOPIC TWO COMPOUND INTEREST.

1. How much do you owe on \$2000 worth of credit card purchases for five years at 18% compounded monthly? \$4886
2. How much do you have in stocks after 40 years, if your investment of \$1000 compounds at 6% annually \$10,286.
3. If you lose 6% a year, what would your \$1000 reduce to over 40 years ? \$129.
4. If you double \$1000 ten times , how much money do you have? \$1,024,000.
5. How many years would it take to double \$1000 at 2% compound interest compounded annually? 35 years.

TOPIC THREE- FUTURE VALUE ANNUITY

Suppose you deposit \$1000 at the end of each year for six years. Annual interest compounded once a year =10%. How much is in your account

(ordinary annuity) at the end of six years? Day 1 you have 0. At the end of year one you put in \$1000. At the end of year two you put in \$1000, etc. .At the end of six years, use the formula for future value S of an annuity. R is the regular payment, i=interest rate, n = number of periods.

$$S = R ((1+i)^n - 1)/i$$

$$\text{So, } S = \$1000((1+.1)^6 - 1)/.10 = 1000(.77)/.1 = \$7700$$

PRACTICE

1. Find the future value of an ordinary annuity if you invest \$1000 at 6% annual interest compounded once for four years. \$4375
2. Find the future value of an annuity with \$10,000 regular payment over five years with interest rate of 3% compounded once annually. \$53,091

ANNUITIES DUE

Suppose we develop an annuity where payment is due at the beginning of each time period. The slight adjusted formula becomes:

$$S = r((1+i)^{(n+1)} - 1)/i - R$$

3. Find the future value of the annuity due with one annual compound of interest if R=\$1200, i=6%, 8 periods. \$12,590.
4. Find the future value of an annuity due with R=\$8000, i=5%, and 7 time periods. \$68,393

5. Find the future value of annuity due with $R=\$5000$, $i=3\%$, n time periods $=20$. \$138,382.

THE TI 83 Plus or 84 Plus easily computes the future values of annuities. Simply enter Beginning for Annuity Due computations End for Ordinary Annuities.

4

6/6

Kristen N. Smith

Dr. Loase

Probability and Statics

September 3, 2014

Reflection

Mathematics is an important aspect in people's lives. I remember when I was in fifth grade and I was taking a math class it felt like the hardest class on earth. As a child, math was never really my subject, but if anyone applies themselves the right way math can be a positive accomplishment in someone's life. While I attended high school I continued to struggle in math, but I did my best to apply myself and remember that it was important to have under my belt. Dr. Loase first article, "Statistics: A key to student Success in College and Life" was a very inspiring article to read. Dr. Loase spoke about Statics being one of the most important classes a college student should take. Math is one of the more difficult courses, especially if someone was not gifted in doing math. Dr. Loase made a very good point that people should not be held back for not understanding the full course of statistics.

In the article Dr. Loase also speaks about the importance of the TI-83 calculator. When I was in high school I started to use the TI-84 and then I realize how important the calculator is. I agree with Dr. Loase statement that many students will never learn elementary algebra, even though they try. Dr. Loase states, "Yes, some students may earn a C in statistics without having mastered addition of fractions. But that student now has an improved chance of graduating from college, earning a decent living, and hiring an accountant to do his or her taxes." This part of the article I read was my favorite because there is hope after obtaining a C in Probability and Statistics. Receiving a C does not cripple any one person to being doomed

from graduation on time or even not having a successful future. The facts and research Dr. Loase gives in both of his articles are interesting as well as inspiring.

In the second article one of the details I found to be the most important part of the article is "Professors need to develop alternative routes to certify that students can succeed in their future profession"(pr.7). Having the opportunity to graduate from college without worrying about a math grade makes college students a lot happier. Dr. Loase is an extremely smart man who I plan on listening to because he gives a lot of great advice. The amount of books Dr. Loase has read makes him a man full of plenty of wisdom. Wisdom should be kept and not thrown away. His articles were very inspiring to become a better student in all my classes. Something else I learned by reading Dr. Loase's articles was to never give up because nothing should stop you from being able to accomplish something great in life.

6 9/10 A MASTERPIECE
AND several
Comments on SURVEY!

Dr. Loase

Report 1- Man's Search for Meaning

22 Feb 08

I found Viktor Frankl's book, Man's Search for Meaning to be both eye- and mind-opening and admire how he conducted himself to get to where he was at the time of the writing of the book. I, myself, have been practicing logotherapy for years, without knowing its proper title. I grew up unhappy and virtually friendless, in a single-parent home (my father had left my mother after she gave birth to her fourth child- which wasn't his). My childhood was composed of taunts from my peers for having an "insane" parent, living with her through all her unstable relationships, and then finally moving out to my father's house, where I felt a little better, but not so much better that I did not try to off myself. Granted, that is nothing like being in a concentration camp, but it was still a difficult time for me that I prefer to forget.

After some time in therapy, I began to live by Frankl's principles. I lived solely to try to find meaning in my life. I have not found anything yet, beyond what I mean to other people. I have not found a cure for cancer or AIDS, nor have I discovered a way to establish a lasting peace between Israel and Palestine. All I have done so far is try to keep the other people in my life together, telling them that there has to be some sort of reason we are all alive, rather than that God was bored enough to make us and is now bored with us, and never admitting that I don't really believe there is any meaning to life at all. I am ignorant, though; just because I have found no personal meaning yet does not mean there is none. I am just looking in the wrong places and am unsure as to how to get to the right ones, or where it is.

My career path looks uncertain right now. I really have no clue what exactly to do with myself. When I first came to Concordia, I wanted to become an international studies major, to learn languages and translate literature and play interpreter for the embassy or the UN. Two and a half years later, after

all the people I have encountered and worked with, I have discovered that I really love teaching other people. Nothing makes me smile larger than when I see one of the students I tutor bring me their first A. I dwelled on that some in my free time- if I love doing something so much, then why am I trying to do something else? Why am I trying to immerse myself in Mandarin and Latin and Spanish and French and Italian and German, confusing my mind on a daily basis, when I know that the bite-size morsels that Concordia offers will only confuse me more? Why am I trying to collect all these small fragments of language and culture and dropping bits and pieces of them as I go along, when there is a bigger picture? What I am learning now is like Hansel's and Gretel's bread-crumbs trail. The small animals that are my lack of motivation and forgetfulness are trailing behind me as I mentally wander through all those foreign locales that I probably will not see, and devouring the little crumbs of what I discover almost as fast as I drop them.

After all my thinking and brooding (complete with pacing to-and-fro), I think I discovered what I am meant to do. French and Italian are rival lovers to me. I want to take at least one of the two tongues and master it. I want to make it bow down and submit to me.

And then I want to show others how to feel the love for it that I do- with the aid of many books and games and movies, of course. I believe that I am meant to teach.

Graham, Illeyana

STAT 121

December 5, 2013

Research Paper

Problem: We wish to determine whether there is a significant difference between males and females who prefer hot beverages (i.e. hot chocolate, coffee, etc.).

Method: With use of a simple two-question survey, I asked students on the Concordia College-NY campus what their beverage prefer was. The two-question survey asked:

1. What gender do you identify with?
2. Do you generally choose hot beverages (i.e. hot chocolate, coffee, etc.) more than cold beverages (juice, water, etc.)?

Tools: The Two Sample Difference of Proportions was used to conduct this research. Pooled Proportion

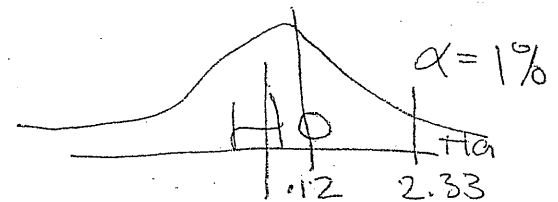
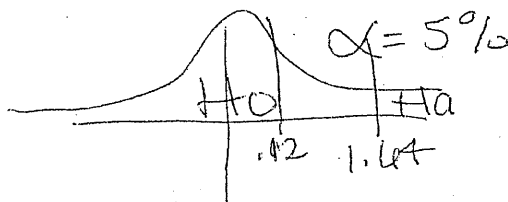
$$P_1: \text{men: } \frac{31}{66} = .4696 \quad P_2: \text{women: } \frac{33}{75} = .44 \quad \bar{P}: \frac{31 + 33}{66 + 75} = \frac{64}{141} = .4539 \approx .45$$

$$z = \frac{P_1 - P_2}{\sqrt{\frac{\bar{P}(1-\bar{P})}{n_1} + \frac{\bar{P}(1-\bar{P})}{n_2}}} = \frac{.45 - .44}{\sqrt{\frac{(.45)(1-.45)}{66} + \frac{(.45)(1-.45)}{75}}} = \frac{.01}{\sqrt{\frac{.2475}{66} + \frac{.2475}{75}}}$$

$$= \frac{.01}{\sqrt{.00375 + .0033}} = \frac{.01}{.08396} = .119 \approx .12$$

Conclusion:

Accept H_0 , $P_1 = P_2$, $\alpha = 5\%$. There is no significant difference in the preference of hot beverages between men and women.



Same for $\alpha = 1\%$. Accept H_0 , $P_1 = P_2$, $\alpha = 1\%$.

Recommendation: A study can be conducted on a larger sample and even on a nation level. Also, one may consider the time at which the research is conducted because hot beverages may be chosen more during the winter season versus the summer season. It is just as important to consider the region as well, begin that the temperatures in some locations vary greatly.